



WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 5th Semester Examination, 2022-23

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any *five* questions from the following:

2×5 = 10

(a) Is the set of vectors $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ are linearly dependent? Justify your answer.

1+1

(b) What is the geometric meaning of the given transformation

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(c) Is any straight line passing through $(0, 0, 0)$ in \mathbb{R}^3 a sub space of \mathbb{R}^3 ? Give reason.

(d) Write down the matrix form of the system of equations

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

(e) Is the vectors $(1, 2)$ and $(-1, 2)$ and linearly independent in \mathbb{R}^3 ? Justify.

(f) Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(g) For what values of k the three vectors $(1, 2, 2)$, $(k, 1, 2)$ and $(2, 2, 1)$ are linearly independent?

(h) Write the standard basis of \mathbb{R}^2 and \mathbb{R}^3 .

(i) Prove that $S = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ is a subspace of \mathbb{R}^3 .

2. (a) If u and v are linearly independent vectors in a vector space V then show that so are $u + v$ and $u - v$.

2

(b) Examine whether the set of vectors are linearly independent in \mathbb{R}^3 .

4

$$\{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}.$$

(c) Define Dilation and Rotation.

2

3. (a) Let A be a singular matrix. Is 0 an eigen value of A ? Justify your answer. 4
 (b) If λ be an eigen value of a non-singular matrix A , then λ^{-1} is an eigen value of A^{-1} . 4
4. (a) Define Eigen space and invariant space with examples. 2+2
 (b) Show that the matrix $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ is not diagonalisable. 4
5. (a) Define a basis of a vector space. Do the vectors $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 2, 1)$ are form a basis of \mathbb{R}^3 ? Justify. 4
 (b) Find the eigen values and corresponding eigen vectors of the following real matrix. 4
- $$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$
6. (a) Prove or disprove: The set $\{(x, y, z) \in \mathbb{R}^3 \mid ax + by + cz = 0 \text{ and } a^2 + b^2 + c^2 \neq 0\}$ is a subspace of \mathbb{R}^3 . 3
 (b) Use elementary row operations on A to obtain A^{-1} where 5
- $$A = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 6 & 4 & 1 \end{bmatrix}$$
7. (a) Let $A = \begin{bmatrix} 3 & 2 & -6 \\ 0 & -1 & 4 \\ 5 & -2 & 0 \end{bmatrix}$. Verify that $A + A'$ is symmetric and $A - A'$ is skew-symmetric and hence express A as the sum of a symmetric and skew-symmetric matrix. 4
 (b) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then verify that A satisfies its own characteristic equation. 4
 Hence find A^{-1} and A^9 .
8. (a) Find a basis of \mathbb{R}^3 containing the vectors $(1, 1, 0)$ and $(1, 1, 1)$. 3
 (b) Let A, B, C be three square matrices such that $A \neq O$ and $AB = AC$, where O is the null matrix. Does it imply $B = C$? Justify your answer. 3
 (c) Define dimension of a finite dimensional vector space V over the field F . Give example. 2

9. (a) Find the 3 by 3 matrix representations of the following transformations.

- (i) projection of any point on the x - y plane.
- (ii) reflection of any point through the x - y plane.

(b) Determine the rank of $A = \begin{pmatrix} x & 1 & 0 \\ 3 & x-2 & 1 \\ 3(x+1) & 0 & x+1 \end{pmatrix}$, for different values of x .

4

10.(a) Solve by matrix method:

$$x + y + z = 4,$$

$$2x - y + 3z = 1,$$

$$3x + 2y - z = 1.$$

4

(b) Reduce the matrix to fully reduced normal form

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 0 & 4 & 6 \\ 3 & 0 & 7 & 2 \end{pmatrix}.$$

4

—x—